

### Homework 3

1. Find the derivative by the limit process.

$$f(x) = \sqrt{x + 4}$$

2. Find the derivative of the function.

(a)

$$f(x) = x^2 - \frac{4}{x^3}$$

(b)

$$f(x) = \frac{1}{x} - 12\sec x$$

(c)

$$f(x) = -2\sqrt{3x^2 + 7}$$

Sol :

1.

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x + 4} - \sqrt{x + 4}}{\Delta x} \cdot \left( \frac{\sqrt{x + \Delta x + 4} + \sqrt{x + 4}}{\sqrt{x + \Delta x + 4} + \sqrt{x + 4}} \right) \\ &= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x + 4) - (x + 4)}{\Delta x [\sqrt{x + \Delta x + 4} + \sqrt{x + 4}]} \\ &= \lim_{\Delta x \rightarrow 0} \frac{1}{\sqrt{x + \Delta x + 4} + \sqrt{x + 4}} \\ &= \frac{1}{\sqrt{x + 4} + \sqrt{x + 4}} \\ &= \frac{1}{2\sqrt{x + 4}} \end{aligned}$$

2.

(a)

$$\begin{aligned} f(x) &= x^2 - \frac{4}{x^3} = x^2 - 4x^{-3} \\ f'(x) &= 2x + 12x^{-4} = 2x + \frac{12}{x^4} \end{aligned}$$

(b)

$$\begin{aligned} f(x) &= \frac{1}{x} - 12 \sec x = x^{-1} - 12 \sec x \\ f'(x) &= -x^{-2} - 12 \sec x \tan x \\ &= \frac{-1}{x^2} - 12 \sec x \tan x \end{aligned}$$

(c)

$$\begin{aligned} f(x) &= -2\sqrt{3x^2 + 7} \\ f'(x) &= -2(3x^2 + 7)^{\frac{1}{2}} \\ &= (-2) \left( \frac{1}{2} \right) (3x^2 + 7)^{-\frac{1}{2}} (6x) \\ &= -\frac{6x}{\sqrt{3x^2 + 7}} \end{aligned}$$